Set Reconfiguration & First-Order Logic

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The goal of the internship is to study logically-defined reconfiguration problems on graphs, design efficient algorithms, and prove complexity lower bounds.

1 Context

A reconfiguration problem asks, given two solutions S and S' to a combinatorial problem, whether one can change S in a step-by-step manner in order to reach S' while ensuring that the transformed set remains a solution. For example, in an undirected graph, given a set of k tokens placed on nonadjacent vertices of the graphs, can one move the tokens one by one to reach a desired configuration, while never having two tokens placed on adjacent vertices?



Example: Following the edges, can we move the red tokens to the green vertices, maintaining non-adjacency?

Reconfiguration problems have been studied on graphs for various combinatorial problems, where the set of tokens must satisfy properties such as: independent set, vertex cover, or dominating set. In most cases, the reconfiguration problem is hard (often PSPACE-complete), it is therefore interesting to study whether restricting the graph, on which the tokens are placed, can improve the running time of the algorithms. More information can be found in this recent survey [BMNS22].

2 Goals

Instead of solving problems one by one, it is interesting to have results of the forms "Using this technique, we can solved this family of problems". Such results are called *meta-theorems*. Meta-theorems must involve a way to describe the family of problems that is being solved. This is achieved trough the use of *logics*. For example, many meta-theorems have been developed for the *model-checking* problem [Gro08]. However, few meta-theorems have been obtained for reconfiguration problems and very little is known, outside of graph classes with bounded tree-width [GIKO22, MNRW14].

The first goal is then to develop meta-theorems for reconfiguration problems that can be defined in first-order logic. This would yield a better understanding of reconfiguration algorithms, specially on sparse graph classes.

A second goal is the study of reconfiguration problems for directed graph classes. While many classes of undirected graphs have been studied in the setting of reconfiguration, very little attention has been given to the case of directed graphs [IIK⁺22]. There are many definitions of width measures for directed graph classes that mirror tree-width in the directed world: directed treewidth, DAG-width, Kelly-width, D-width, DAG-depth, etc. The precise definitions for all of these notions can be found in the following book chapter [KK18].

The studying of reconfiguration problems over directed graph classes would bring a deeper understanding of the algorithmic properties of the various width measure presented above.

3 Context and advisors

The internship is to be performed in LIMOS Laboratory, located in Clermont-Ferrand, France.

The intern will be supervised by Alexandre Vigny and Mamadou Kanté, both members of LIMOS, and the AlCoLoco research group. This research group has been growing rapidly with the recruitment of several new researchers in the past couple of years.

4 How to apply

Send your applications by e-mail to: alexandre.vigny@uca.fr. Applications must contain

- a CV and,
- one recommendation letter, or the email address of someone that can recommend you.

A significant interest in the fields of logic, algorithmic, and/or graph theory is expected. There is no strict deadline for the applications but the sooner it comes, the greater is the probability to be selected. For more information, do not hesitate to send an email to alexandre.vigny@uca.fr

References

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